

# A level Mathematics

3 exams in May/June of year 13

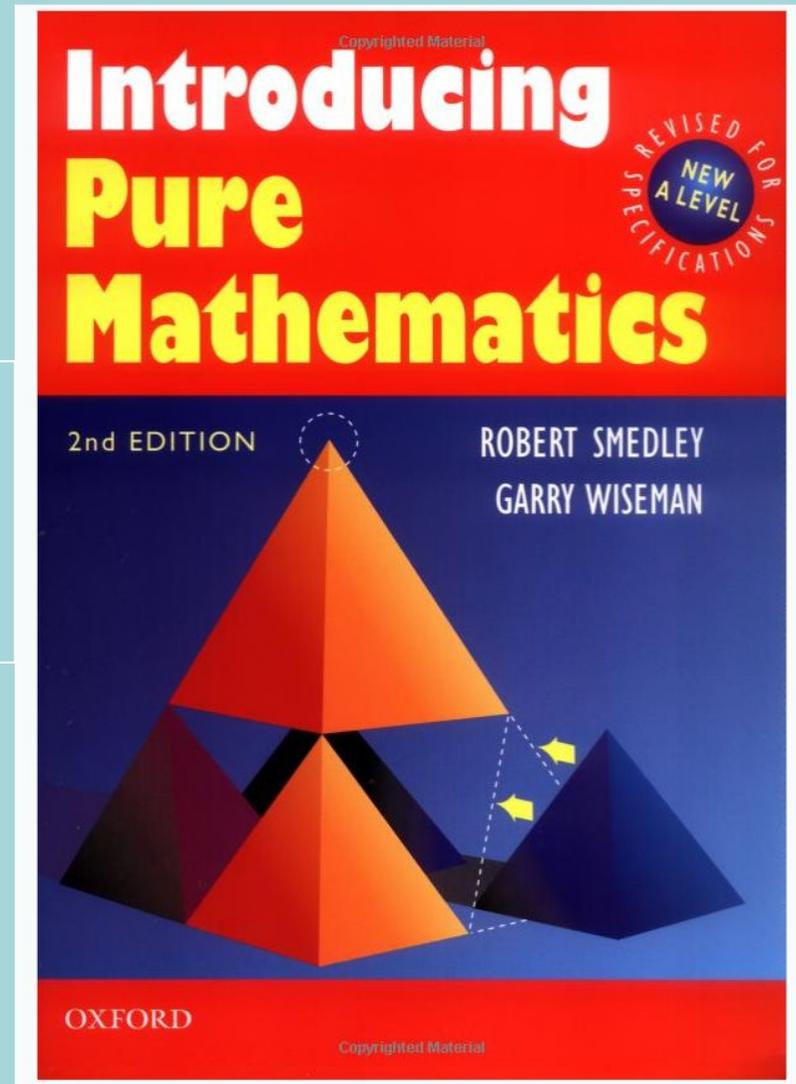
each lasting 2 hours

- Pure Mathematics 1
- Pure Mathematics 2
- Statistics and Mechanics

# Pure Mathematics

Text Book

Borrow from school  
£10 deposit

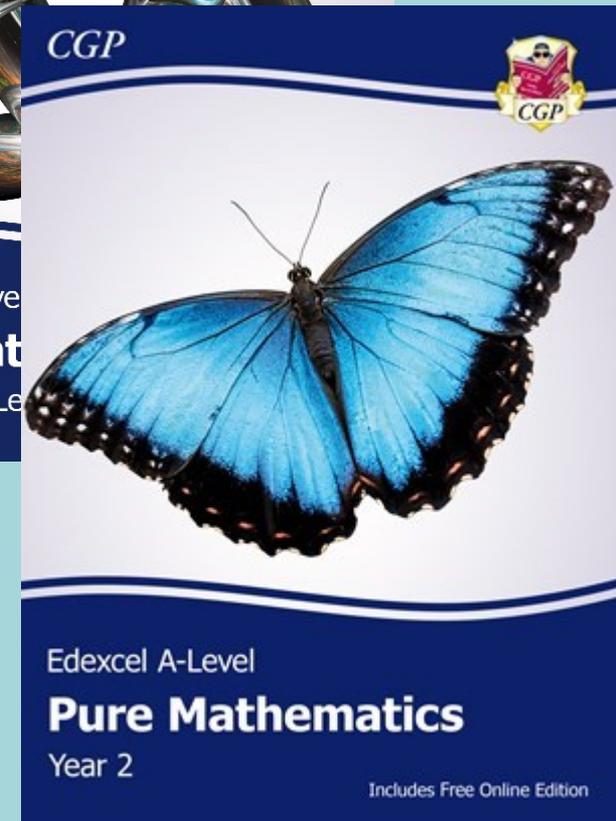


# Pure Mathematics

## Text Books

Buy through school  
in September

Cost: about £11

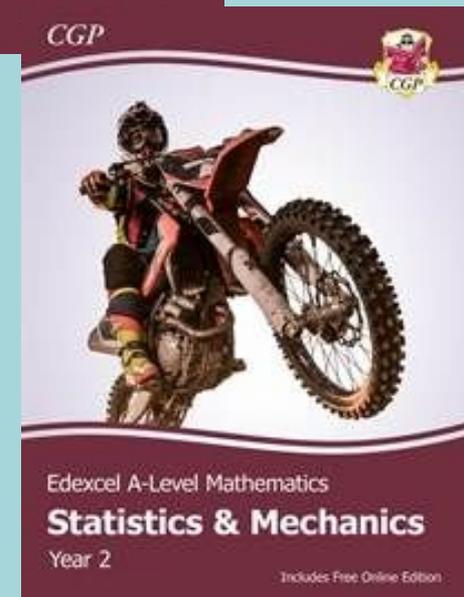
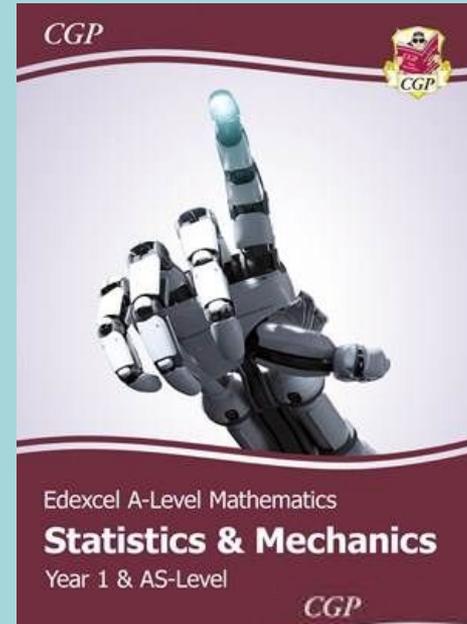


# Statistics and Mechanics

Text Books

Buy yourself

Cost: about £6 each





# A level Further Mathematics

4 exams in May/June of year 13  
each lasting  $1\frac{1}{2}$  hours

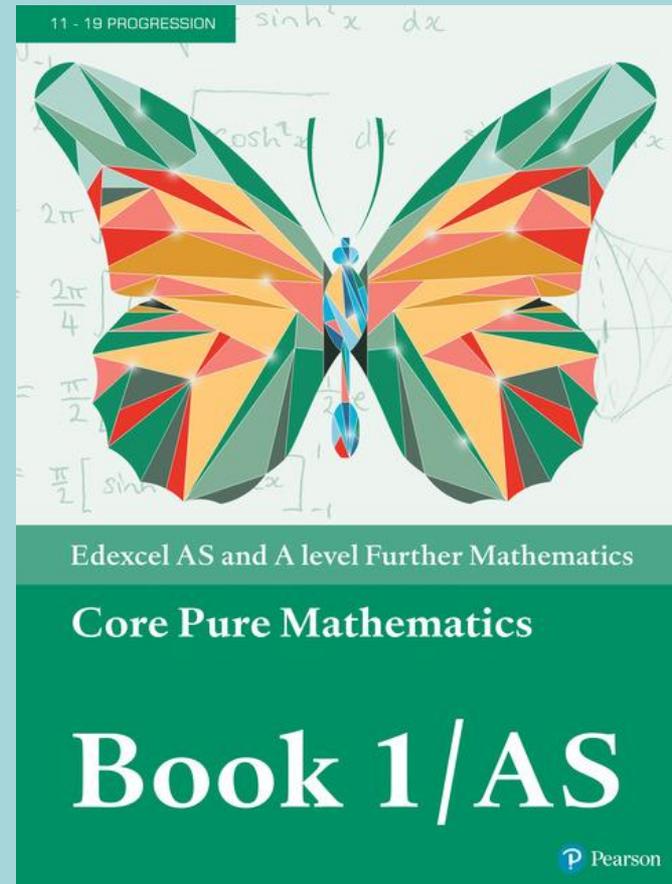
- Pure Mathematics 1
- Pure Mathematics 2
- Further Statistics
- Further Mechanics

# Further Pure Mathematics

## Text Books

Buy through school  
in September

Cost: about £24



# Advanced Maths Support Programme



## Transition to A Level Mathematics

We have arranged access to this course to help prepare and support you as you start your A level journey. The modules give you an insight into the level of thinking required at A Level and an opportunity to develop your mathematical knowledge in preparation for the course ahead.

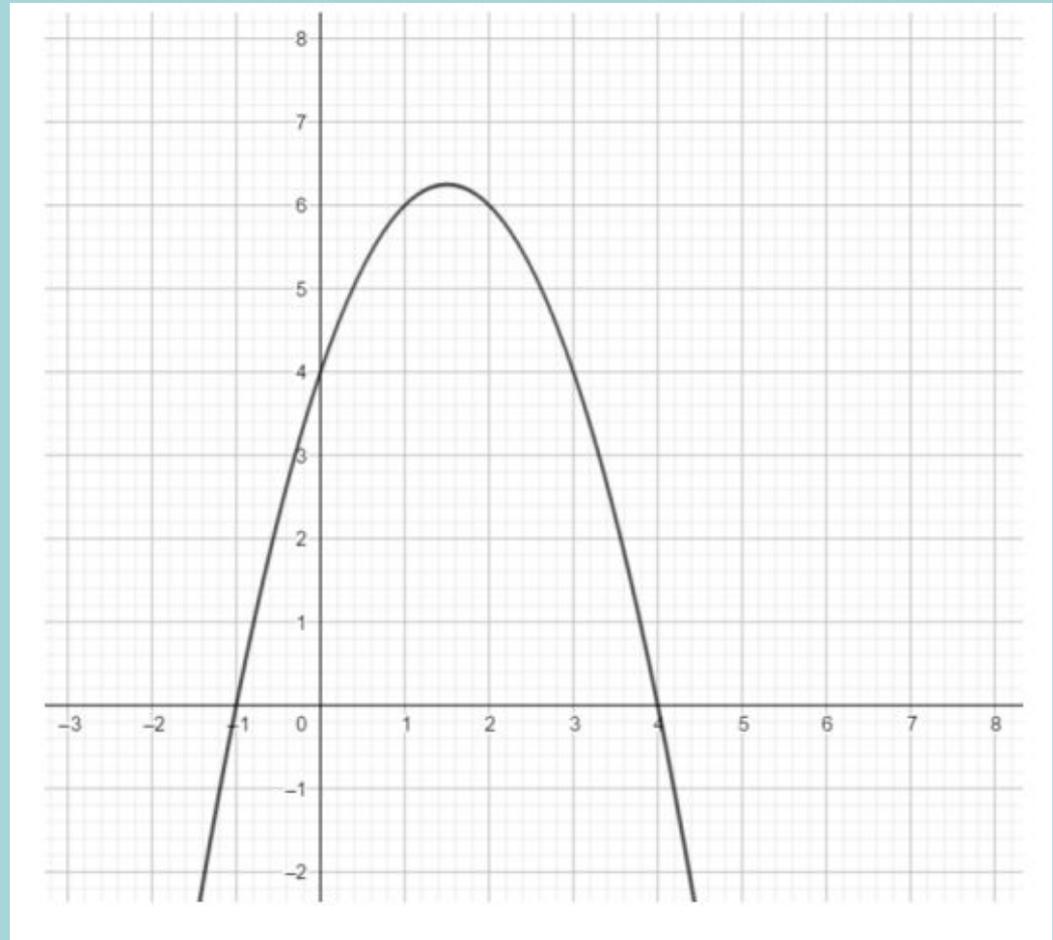
<https://my.integralmaths.org/login/index.php>

Your username and password will be given to you through Google Classroom, or emailed for external candidates.

You will be asked to complete **at least** the Algebraic Manipulation section and print the certificate to show us in September. The more you do the better prepared you will be. Enjoy exploring these resources!

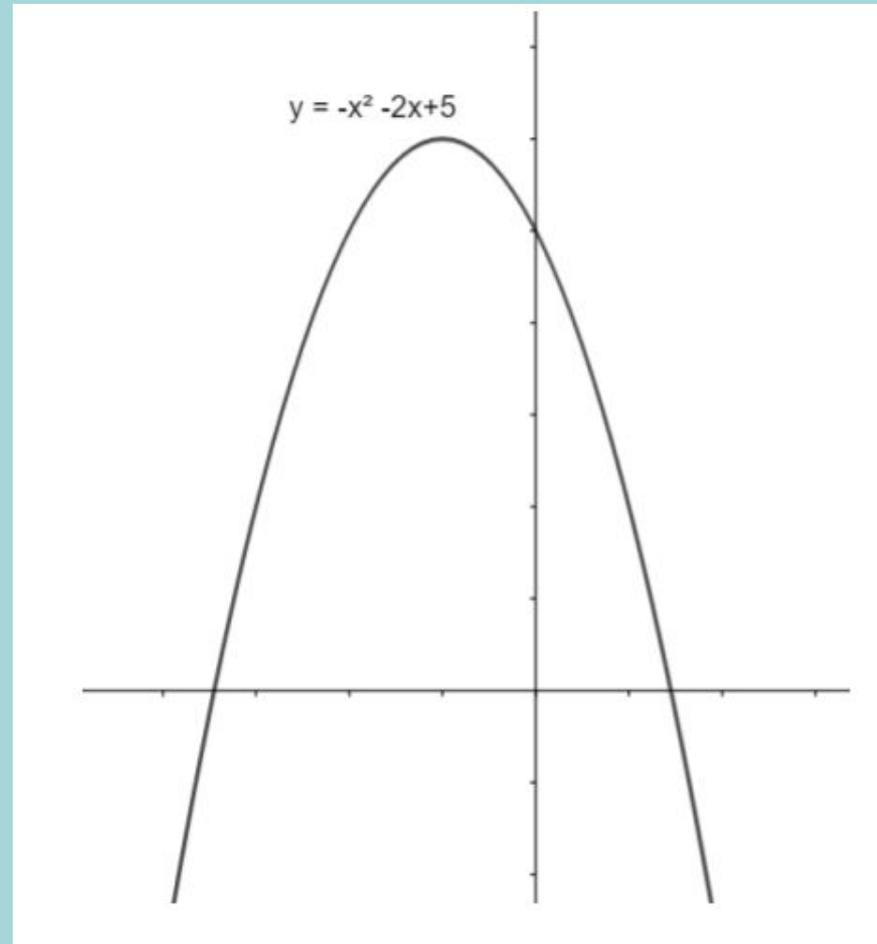
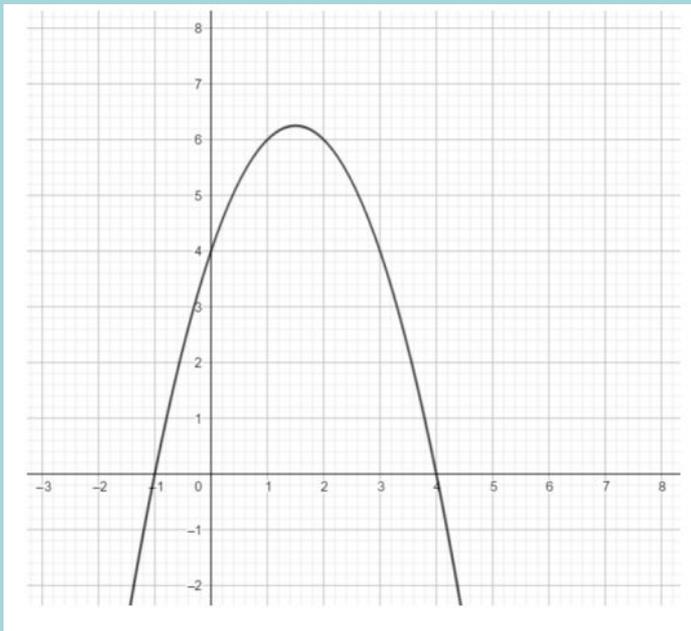
## Graphs

What might you be asked about this diagram?



## Graphs

What might you be asked about this diagram?  
How is it different to the diagram before?



## Graphs & Areas

This is an actual exam question.

What is different to GCSE?

7.

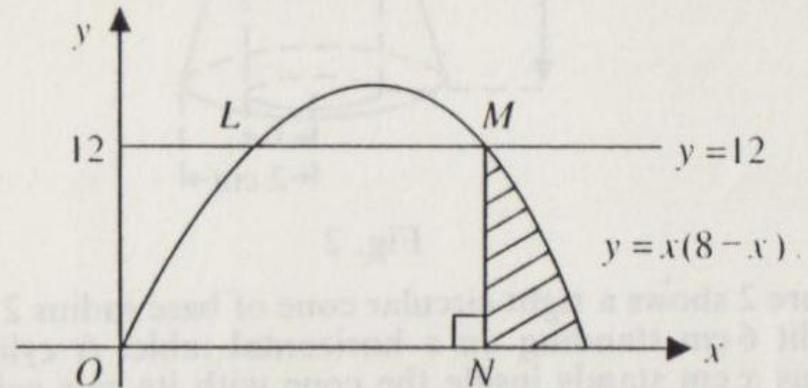


Fig. 1

Figure 1 shows the curve  $C$  with equation  $y = x(8 - x)$  and the line with equation  $y = 12$  which meet at the points  $L$  and  $M$ .

(a) Determine the coordinates of the point  $M$ .

Given that  $N$  is the foot of the perpendicular from  $M$  onto the  $x$ -axis,

(b) calculate the area of the shaded region which is bounded by  $NM$ , the curve  $C$  and the  $x$ -axis.

(10 marks)

## Graphs & Gradients

10.

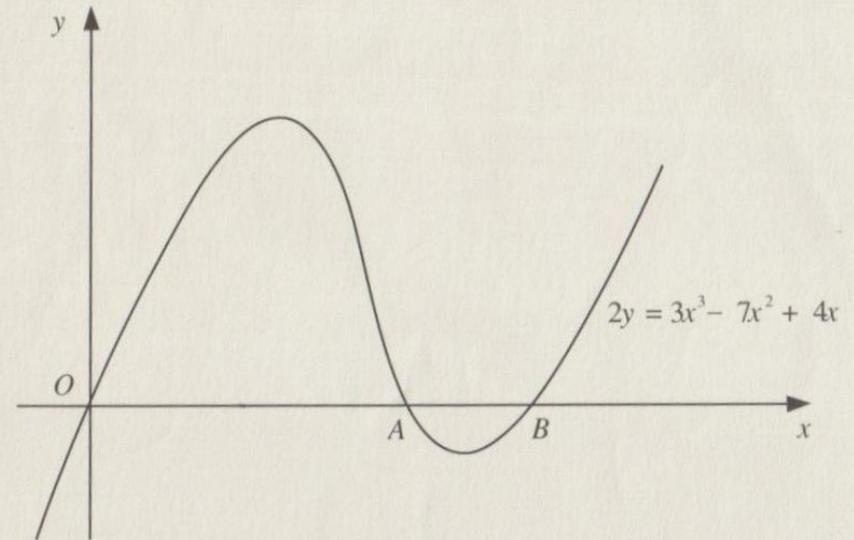


Fig. 2

Figure 2 shows a sketch of part of the curve  $C$  with equation

$$2y = 3x^3 - 7x^2 + 4x$$

which meets the  $x$ -axis at the origin  $O$ , the point  $A(1,0)$  and the point  $B$ .

(a) Find the coordinates of  $B$ .

The normals to the curve  $C$  at the points  $O$  and  $A$  meet at the point  $N$ .

(b) Find the coordinates of  $N$ .

(c) Calculate the area of  $\triangle OAN$ .

## Describing and Comparing Data Sets

The mark  $x$  obtained by each of 45 students randomly selected from those students who sat the accountancy examination was recorded. The stem and leaf diagram below summarises the marks.

Mark	(5   3 means 53)	Totals
5	0 1 3 3 4 4	(6)
5	5 6 7 9	(4)
6	1 1 3 3 4 4 4	(7)
6	5 7 8 8 9	(5)
7	3 3 4 4 4 4	(6)
7	5 5 6 6 7 7 7 7 8 8 8 9 9	(13)
8	0 0 1 1	(4)

(b) Using graph paper and showing your scale clearly, construct a box plot to represent these data. (8 marks)

(c) Comment on the skewness of this distribution. (1 mark)

For the above sample,  $\Sigma x = 3085$  and  $\Sigma x^2 = 215\,569$ .

(d) Find the mean and the standard deviation of this sample of marks. (3 marks)

The mean and the standard deviation of the marks of all the students who sat the examination were 65 and 16.5 respectively. The examiners decided that the mark of each student should be scaled by having 10 marks subtracted and then reduced by a further 10%.

(e) Find the mean and the standard deviation of the scaled marks. (5 marks)

## Probability

1. The events  $A$  and  $B$  are mutually exclusive. Given that

$P(A) = 0.4$  and  $P(B) = 0.5$  find

(a)  $P(A \cap B)$ , (1 mark)

(b)  $P(A \cup B)$ , (2 marks)

(c)  $P(A' \cap B)$ . (2 marks)

## Probability

4. There are 60 students in the sixth form of a certain school. Mathematics is studied by 27 of them, biology by 20 and 22 students study neither mathematics nor biology.
- (a) Find the probability that a randomly selected student studies both mathematics and biology.
- (b) Find the probability that a randomly selected mathematics student does not study biology.
- A student is selected at random.
- (c) Determine whether the event “studying mathematics” is statistically independent of the event “not studying biology”.
- (10 marks)**

## Probability Distributions

6. A factory makes steel rods and steel tubes. The internal diameter  $X$  of a steel tube is normally distributed with mean 3.50 cm and standard deviation 0.03 cm.

(a) Find the probability that a randomly selected tube has an internal diameter less than 3.48 cm. (3 marks)

The diameter  $Y$  of a steel rod is normally distributed with mean 3.45 cm and standard deviation 0.03 cm.

(b) Find the probability that a randomly selected rod has a diameter greater than 3.40 cm and less than 3.51 cm. (3 marks)

## Hypothesis Testing

In a manufacturing process 25% of articles are thought to be defective. Articles are produced in batches of 20

- (a) A batch is selected at random. Using a 5% significance level, find the critical region for a two tailed test that the probability of an article chosen at random being defective is 0.25

You should state the probability in each tail which should be as close as possible to 0.025

(5)

The manufacturer changes the production process to try to reduce the number of defective articles. She then chooses a batch at random and discovers there are 3 defective articles.

- (b) Test at the 5% level of significance whether or not there is evidence that the changes to the process have reduced the percentage of defective articles. State your hypotheses clearly.

(5)

## Sampling

**4** A manufacturer of microswitches is testing the reliability of its switches. It uses a special machine to switch them on and off until they break.

**a** Give one reason why the manufacturer should use a sample rather than a census.

The company tests a sample of 10 switches, and obtains the following results:

23 150    25 071    19 480    22 921    7 455

**b** The company claims that its switches can be operated an average of 20 000 times without breaking. Use the sample data above to comment on this claim.

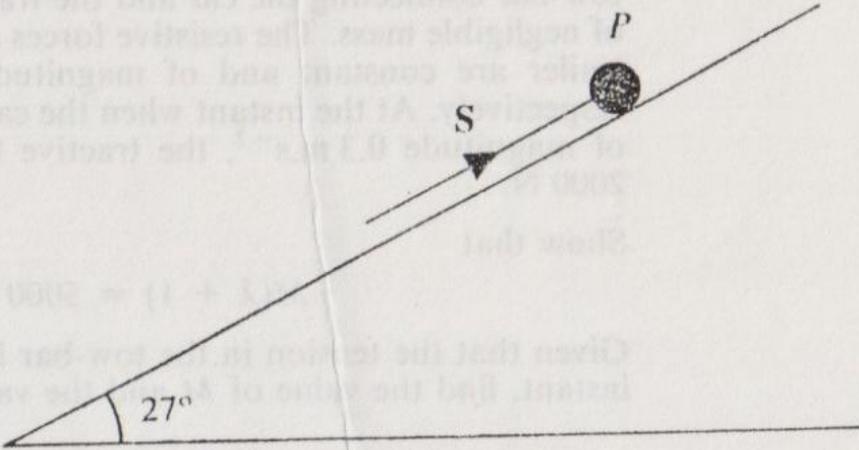
**c** Suggest one way the company could improve its prediction.

## Large Data set

1 CAMBORNE		© Crown Copyright Met Office 2015								
2 NGR = 1627E 4067N										
3 Altitude = 87 metres										
4 Latitude = 50:22N Longitude = 05:33W										
5										
6	Date	Daily Mean Temperature (0900-0900) (°C)	Daily Total Rainfall (0900-0900) (mm)	Daily Total Sunshine (0000-2400) (hrs)	Daily Mean Windspeed (0000-2400) (kn)	Daily Mean Windspeed (0000-2400) (Beaufort conversion)	Daily Maximum Gust (0000-2400) (kn)	Daily Maximum Relative Humidity %	Daily Mean Total Cloud (oktas)	Daily Mean Visibility (Dm)
23	17/05/1987	9.7	tr	0	7	Light	19	93	8	2900
24	18/05/1987	10.4	0	4.1	14	Moderate	27	86	6	2300
25	19/05/1987	9.5	0	4.4	8	Light	17	96	4	1900
26	20/05/1987	11.1	0	11.9	6	Light	15	99	3	1600
27	21/05/1987	10.5	0	11.3	12	Moderate	26	87	4	2700
28	22/05/1987	11.1	0.3	7.8	11	Moderate	27	87	6	2600
29	23/05/1987	9.8	0	10.8	11	Moderate	27	87	4	1400
30	24/05/1987	10.2	4.2	5.1	16	Moderate	32	94	6	1000
31	25/05/1987	12.6	1.7	10.7	13	Moderate	35	86	6	2500
32	26/05/1987	10.4	7.5	0	7	Light	22	95	7	1500
33	27/05/1987	11.3	0.1	0	9	Light	21	98	8	500
34	28/05/1987	12.1	0	5.5	8	Light	19	96	6	1600
35	29/05/1987	12.0	tr	0	7	Light	20	99	7	1500
36	30/05/1987	11.6	7.1	0	11	Moderate	28	99	7	900
37	31/05/1987	12.7	0	8.6	9	Light	19	98	7	1400
38	01/06/1987	12.7	9	0.1	6	Light	16	94	7	1900
39	02/06/1987	12.8	3.9	1.4	13	Moderate	29	99	8	1100
40	03/06/1987	13.5	3.1	4.1	12	Moderate	25	98	7	1300
...	04/06/1987	13.5	0.1	6.6	6	Light	14	96	6	1600

## Objects being pushed & pulled

6.



The diagram shows a right-angled triangle representing an inclined plane. The angle between the horizontal base and the inclined side is labeled  $27^\circ$ . A small shaded circle representing a parcel  $P$  is positioned on the inclined side. A force vector  $S$ , represented by a triangle with a tail, is shown acting on the parcel  $P$  and pointing up the incline. A line is drawn parallel to the inclined side, passing through the tail of force  $S$ .

Fig. 1

A small parcel  $P$ , of mass  $1.5 \text{ kg}$ , is placed on a rough plane inclined at an angle of  $27^\circ$  to the horizontal. The coefficient of friction between the parcel and the plane is  $0.3$ . A force  $S$ , of variable magnitude, is applied to the parcel as shown in Fig. 1. The line of action of  $S$  is parallel to a line of greatest slope of the inclined plane.

Determine, in  $\text{N}$  to 1 decimal place, the magnitude of  $S$  when the parcel  $P$  is in limiting equilibrium and on the point of moving

(a) down the plane,

(b) up the plane.

(12 marks)

## Vectors

2. Two horizontal forces, **P** and **Q**, act on a particle.  
The force **P** is of magnitude 8 N and acts in the direction whose bearing is  $330^\circ$ .  
The force **Q** is of magnitude 15 N and acts in the direction whose bearing is  $060^\circ$ .  
Calculate the magnitude and the direction of the resultant of **P** and **Q**, giving the direction as a bearing to the nearest degree. **(6 marks)**

## Travel Graphs

3. A car is travelling along a straight motorway at a constant speed  $V \text{ m s}^{-1}$ . Ten seconds after passing a speed-limit sign, the driver brakes and the car decelerates uniformly for 5 seconds, reducing its speed to  $30 \text{ m s}^{-1}$ .

(a) Sketch a speed-time graph to illustrate this information.

Given that the car covers a distance of 600 m in the 15 second period, find

(b) the value of  $V$ ,

(c) the deceleration of the car.

**(8 marks)**

## Objects in Equilibrium

A beam  $AB$  has length 15 m. The beam rests horizontally in equilibrium on two smooth supports at the points  $P$  and  $Q$ , where  $AP = 2$  m and  $QB = 3$  m. When a child of mass 50 kg stands on the beam at  $A$ , the beam remains in equilibrium and is on the point of tilting about  $P$ . When the same child of mass 50 kg stands on the beam at  $B$ , the beam remains in equilibrium and is on the point of tilting about  $Q$ . The child is modelled as a particle and the beam is modelled as a non-uniform rod.

(a) (i) Find the mass of the beam.

(ii) Find the distance of the centre of mass of the beam from  $A$ .

(8)

When the child stands at the point  $X$  on the beam, it remains horizontal and in equilibrium. Given that the reactions at the two supports are equal in magnitude,

(b) find  $AX$ .

(6)

## Velocity and Acceleration

A lorry is moving along a straight horizontal road with constant acceleration. The lorry passes a point  $A$  with speed  $u \text{ m s}^{-1}$ , ( $u < 34$ ), and 10 seconds later passes a point  $B$  with speed  $34 \text{ m s}^{-1}$ . Given that  $AB = 240 \text{ m}$ , find

(a) the value of  $u$ , (3)

(b) the time taken for the lorry to move from  $A$  to the mid-point of  $AB$ . (6)

Sort the cards into  
several categories.

Be ready to discuss  
your choices.

<p><b>A</b></p> <p>If <math>y = x^{\frac{1}{2}} + 8</math>, find <math>y</math> when <math>x = 4</math></p> $y = 4^{\frac{1}{2}} + 8$ $= 2 + 8$ $= 10$	<p><b>B</b></p> $4 - 3x \leq 13$ $-3x \leq 9$ $x \leq -3$
<p><b>C</b></p> $x^2 + 7x + 12 = 6$ $(x + 3)(x + 4) = 6$ $x + 3 = 6 \quad \text{or} \quad x + 4 = 6$ $x = 3 \quad \text{or} \quad x = 2$	<p><b>D</b></p> $6 + 2(x - 5) = 6 + 2x - 10$ $= 2x - 4$
<p><b>E</b></p> $4 - (x + 2) = 4 - x + 2$ $= 6 - x$	<p><b>F</b></p> $(x + y)^2 = x^2 + y^2$
<p><b>G</b></p> $\frac{3}{x^2} = 3x^{-2}$	<p><b>H</b></p> $\frac{x^2 + 3x + 2}{x^2 - 1} = \frac{3x + 2}{-1}$ $= -3x - 2$

<b>I</b>	$x^2 > 16$ $x > 4 \quad \text{or} \quad x < -4$ <p>i.e. <math>-4 &gt; x &gt; 4</math></p>	<b>J</b>	$(x + 2)^2 = x^2 + 4x + 4$
<b>K</b>	$x^2 + 10x + 21 = 0$ $(x + 3)(x + 7) = 0$ $x + 3 = 0 \quad \text{or} \quad x + 7 = 0$ $x = -3 \quad \quad \quad \text{or} \quad x = -7$	<b>L</b>	$\frac{1}{4x} = 4x^{-1}$
<b>M</b>	$2x + 6 \geq 2$ $2x \geq -4$ $x \geq -2$	<b>N</b>	$\frac{x}{1 + x^2} = \frac{x}{1} + \frac{x}{x^2}$ $= x + \frac{1}{x}$
<b>O</b>	<p>If <math>y = x^2 + 3</math>, find <math>y</math> when <math>x = -4</math></p> $y = -4^2 + 3$ $= -16 + 3$ $= -13$	<b>P</b>	$\frac{1 + x^2}{x} = \frac{1}{x} + \frac{x^2}{x}$ $= \frac{1}{x} + x$

These cards were correct:

A D G J

K M P

Can you correct the others?

B

$$4 - 3x \leq 13$$

$$-3x \leq 9$$

$$x \leq -3$$

C

$$x^2 + 7x + 12 = 6$$

$$(x+3)(x+4) = 6$$

$$x+3 = 6$$

$$x = 3$$

or

$$x+4 = 6$$

or

$$x = 2$$

E

$$\begin{aligned}4 - (x + 2) &= 4 - x + 2 \\ &= 6 - x\end{aligned}$$

F

$$(x + y)^2 = x^2 + y^2$$

H

$$\frac{x^2 + 3x + 2}{x^2 - 1} = \frac{3x + 2}{-1}$$
$$= -3x - 2$$

$$I \quad x^2 > 16$$

$$x > 4 \quad \text{or} \quad x < -4$$

$$\text{i.e.} \quad -4 > x > 4$$

L

$$\frac{1}{4x} = 4x^{-1}$$

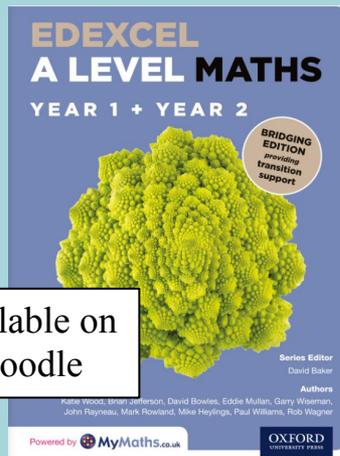
N

$$\frac{x}{1+x^2} = \frac{x}{1} + \frac{x}{x^2}$$
$$= x + \frac{1}{x}$$

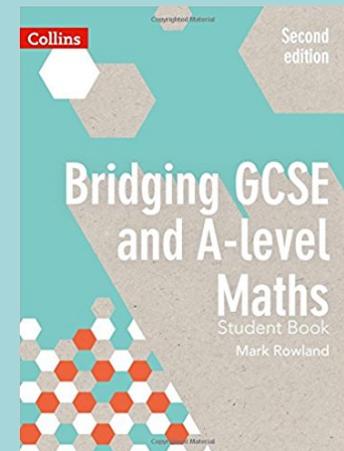
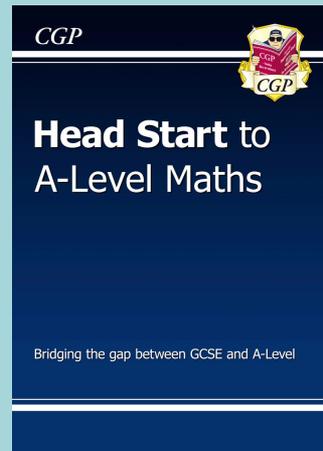
O If  $y = x^2 + 3$ , find  $y$  when  $x = -4$

$$\begin{aligned}y &= -4^2 + 3 \\ &= -16 + 3 \\ &= -13\end{aligned}$$

If you did not notice all of the mistakes, then you might like to use some of the resources specially written to help you to make a good start to A level maths and revise the most important GCSE topics before term starts in September.

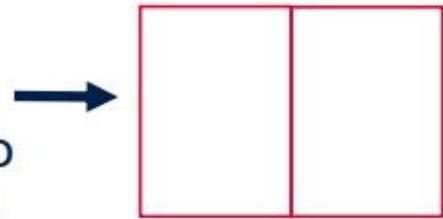


Available on Kerboodle

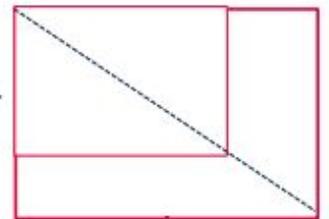




Actually take two sheets and place them side by side to make a piece of A3.



Then place another sheet of A4 on top of it like this:



What do you notice about the ratio of the sides of an A3 sheet compared with the A4 sheet?

Thinking about the ratio of the long side to the short side we get:



$$\frac{x}{1} = \frac{2}{x} \rightarrow x^2 = 2 \rightarrow x = \sqrt{2}$$

Therefore, for A4, A3, A2, etc... the length of the long side divided by the length of the short side is always  $\sqrt{2}$



Read about how **Irrational numbers** can “Inspir-al” you!  
It’s where mathematics and art meet!



Discover the proof, that  $\sqrt{2}$  is irrational – without getting murdered like Hippasus.



Watch this video to find out more about the special properties of A4 paper and discover what makes  $\sqrt{2}$  one of the most popular surds of all time.

# Maths for fun

Follow the links to these three Maths sites and spend some time watching Maths videos and trying out some of the activities. Choose whatever interests you.

<https://www.think-maths.co.uk/>

<https://www.numberphile.com/>

<https://tomrocksmaths.com/>